



Derivation of the 12-Lead Electrocardiogram  
using Abstract factor analysis and simplex optimization

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**Abstract:** The electrocardiogram (ECG) is a lead-vector system that models the electrical activity of the heart. Both dipolar and multipolar factors have been shown to influence cardiac electrical activity. Although many lead placement conventions have been utilized, controversy exists in defining the number of ECG leads needed to adequately describe cardiac electrical activity. Abstract factor analysis (AFA) is a computer modeling technique that can be applied to ECG voltage-time data in order to identify a minimum number of lead-vectors accounting for the variance in the measured ECG data set and also serves to improve error in the measured data. It would be advantageous to derive the ECG from a smaller number of standard measured leads using a universal patient coefficient matrix. Simplex optimization (SOP) is a multivariate nonlinear statistical technique that allows the derivation of a coefficient matrix by varying all lead vectors simultaneously. The objective of this study is to derive the 12-lead standard ECG from a minimum number of lead-vectors identified by AFA, then calculate a universal transformation matrix from a minimum set of leads that comprise the conventional standard 12 lead set using the SOP method. Using the universal transformation matrix calculated from the AFA-SOP technique, 20 test case 12-lead ECGs of variable pathology were derived for comparison to the measured data. All 20 derived ECGs were diagnostically and morphologically accurate using this method.

## INTRODUCTION

The standard ECG record consists of a mathematical set of 12 lead-vector waveforms {I, II, III, aVR, aVL, aVF, V1, V2, V3, V4, V5, V6} read by a physician using a pattern recognition technique. In the usual configuration, 10 electrodes are placed on the body torso to measure the electrical potentials of the standard 12 leads. The conceptual mathematical model as described by Gustafson [1] is as follows:

$$V(t) = \sum_{i=1}^n AX_i(t) + E(t) \quad (1)$$

where  $V(t)$  = voltage-time data array of the standard ECG lead set,  $X$  is a smaller lead-vector set describing the space,  $A$  is a transformation matrix, and  $E(t)$  is an error matrix. AFA [2] attempts

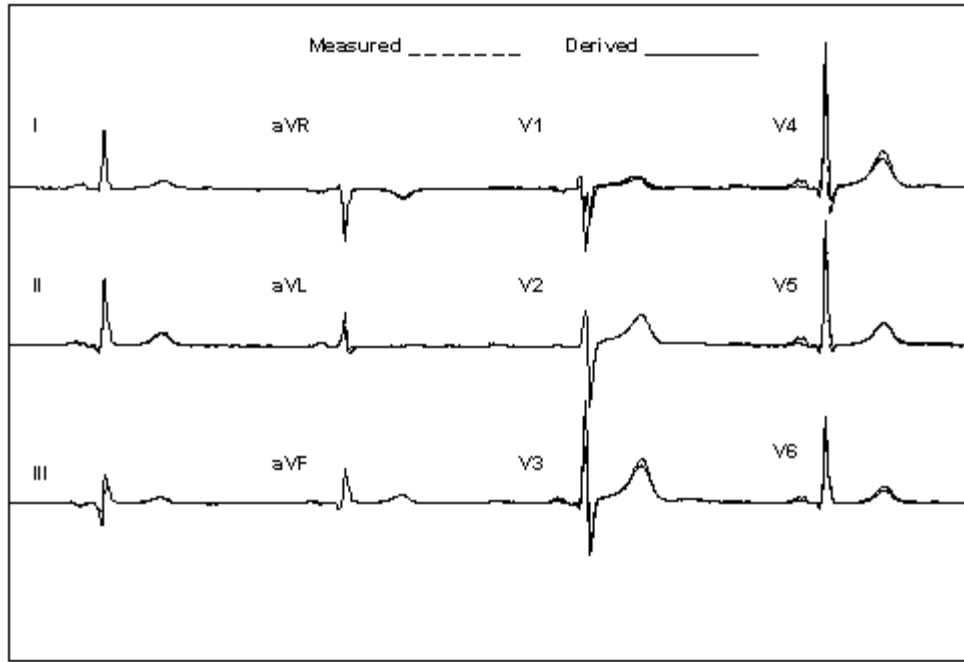
to define  $n$  as the minimum number of lead-vectors needed to span the cardiac electrical space. AFA will also calculate a patient specific transformation matrix  $A$  such that when multiplied by the minimum lead vector set  $X$  will yield an AFA derived 12-lead ECG. The concept of the derived ECG is not new. Dower [3] has reported on deriving the 12-lead ECG from standard vectorcardiographic leads. Dower [4] has also reported on deriving the 12-lead ECG from nonstandard lead-vectors described as the EASI lead set. This study describes the derivation of the 12-lead ECG from a set of 3 leads belonging to the standard 12-lead ECG set. SOP is used to calculate a universal transformation matrix that is independent of gender, pathology, time, and body habitus. In this study, the SOP method described by Shavers [5] was used to derive the 12-lead ECG from the set of leads {I, aVF, V2}.

## **METHODS**

Twenty standard 12-lead ECG recordings were acquired from 5 normal men, 5 normal women, and 5 men and 5 women with acute myocardial infarction (MI). Each ECG was acquired using a Marquette MAC-15 machine that automatically digitized each patient ECG resulting in a  $300 \times 8$  voltage-time data array. The lead set {I, II, V1-V6} was measured and leads III, aVR, aVL, aVF were calculated according to standard formulae. Each test case ECG  $300 \times 12$  data array was subjected to AFA to define the minimum number of lead vectors spanning the space of electrical activity. AFA was also applied to a different training set of 20 ECG cases from which SOP was used to derive a universal transformation matrix. The AFA application to the 20 test cases demonstrated that 3 leads accounted for  $99.16\% \pm 0.22\%$  of the information content in the 12-lead ECG space of electrical activity. Therefore, SOP was applied to the 3 lead set comprised of {I, aVF, V2} such that a  $3 \times 12$  universal transformation matrix was calculated from the AFA derived training set ECG recordings. The 20 test case 12-lead ECGs were derived by multiplying the AFA-SOP matrix  $K$  by the measured lead-vectors {I, aVF, V2}. The derived and measured ECGs were compared for diagnostic and morphologic correlation.

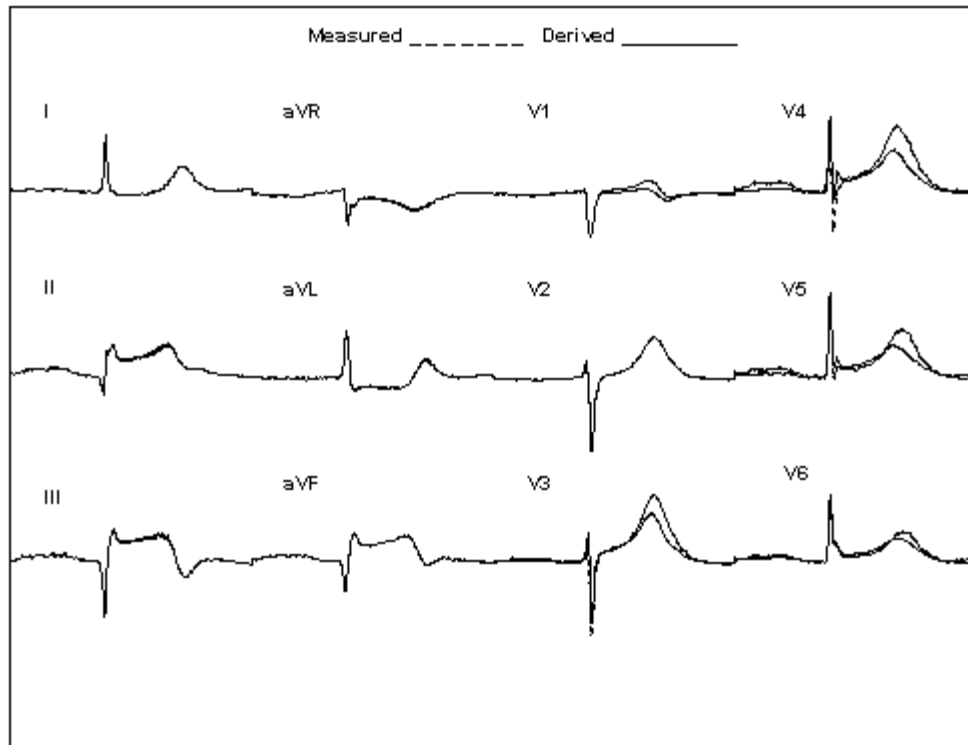
## **RESULTS**

Despite some minor morphologic changes, all 20 derived ECGs were diagnosed correctly when compared to the measured 12-lead standard ECGs. An example of a normal male AFA-SOP derived ECG compared to the measured ECG is shown in Figure 1.



*Figure 1. Normal male ECG*

An example of a female acute MI derived ECG compared to the AFA derived and measured ECG is shown in Figure 2.



*Figure 2. Female acute MI ECG*

## **DISCUSSION**

This small methodology study is a work in progress that demonstrates a standard 12-lead ECG can be derived from a set of 3 leads belonging to the standard 12-lead set. This is important because of the familiarity with this lead convention, thus eliminating the need for training and validation of a new lead convention in the clinical setting. The lead set {I, aVF, V2} are minimally variable in their electrode placement so as to minimize error. Since this lead set is almost orthogonal as described by Chou [6], the derived ECG is essentially contained in the spatial 3D vector loop comprised of the {I, aVF, V2} lead set, a representation of which is depicted in Figure 3. In addition, SOP allows the calculation of a universal transformation matrix that is independent of time, gender, pathology, and body habitus. AFA is a data “pretreatment” method that minimizes error commonly seen in measured data examples of which include baseline noise, wandering baseline, and lead placement variability. SOP applied to this AFA improved ECG data results in a universal transformation that in theory should have greater clinical accuracy.

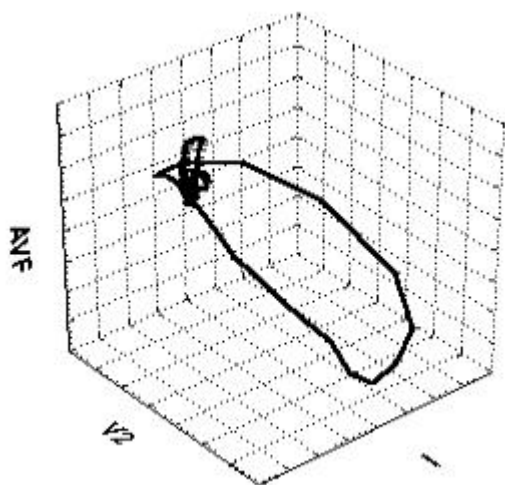


Figure 3. Spatial ECG

The improvements in computer technology now make the clinical application of this technique very appealing. Large matrix calculations of this complexity are now performed in microseconds using current desktop technology. The application of the derived ECG in the setting of emergency medicine and prehospital care are just two of many potential clinical uses of this process. While improvements of this technique will be forthcoming, it is emphasized that clinical validation of this process is warranted. Finally, as skeptics of the derived ECG no doubt exist, we believe that the real question facing clinicians is not “are these ECGs the same?” but rather “which one is clinically correct?”!!!

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